OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4724:** Core Mathematics 4

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question	Answer	Marks	Guidance	
1	$u = x$ and $\mathrm{d}v = \cos 3x$	M1	integration by parts as far as $\mathrm{f}(x) \pm \int \mathrm{g}(x)\,\mathrm{d}x$	Check if labelled *v*,d*u*
	$x \times \frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x \mathrm{d}x$	A2	**A1** for $x \times k \sin 3x - \int k \sin 3x\,\mathrm{d}x$; $k \neq \frac{1}{3}$ or 0	k may be negative
	$\frac{x}{3}\sin 3x + \frac{1}{9}\cos 3x$ [+ c] cao www ISW	A1	Not ….. $\frac{1}{3}\left(\frac{1}{3}\cos 3x\right)$ or ….. $- -\frac{1}{9}\cos 3x$	
		[4]		
2	<u>The first 3 marks refer to the expansion…</u>	……..	<u>of $\left(1 - \frac{16x}{9}\right)^{3/2}$ and to no other expansion</u>	
	First 2 terms = $1 - \frac{8}{3}x$	B1	Allow any equiv fraction for the $-\frac{8}{3}$ and ISW	$\frac{3}{2} . - \frac{16}{9}$ is not an equiv fraction
	3rd term = $\frac{\frac{3}{2}\cdot\frac{1}{2}}{1.2}\left(-\frac{16x}{9}\right)^2$	M1	Allow clear evidence of intention,e.g. $\frac{\frac{3}{2}\cdot\frac{1}{2}}{1.2}\frac{-16x^2}{9}$	
	$= \frac{32}{27}x^2$	A1	Allow any equiv fraction for the $\frac{32}{27}$ and ISW	
	Complete expansion $\approx 27 - 72x + 32x^2$	A1	cao No equivalents. Ignore any further terms	If expansion $(a+b)^n$ used, award B1,B1,B1 for $27, -72x, 32x^2$
	valid for $\frac{-9}{16} < x < \frac{9}{16}$ or $\lvert x\rvert < \frac{9}{16}$	B1	oe Beware, e.g. $x < \left\lvert\frac{9}{16}\right\rvert$	condone $\leq$ instead of $<$
		[5]		

Question			Answer	Marks	Guidance	
3			For attempt at product rule on xy^2	M1	or changing equation to $y^2 = x + x^{-1}$	
			$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$	B1	soi in the differentiating process	
			$\frac{dy}{dx} = \frac{2x - y^2}{2xy}$ or $\frac{1 - x^{-2}}{2y}$	A1	Award B1 for $(\pm)\frac{1}{2}(x + x^{-1})^{-\frac{1}{2}}(1 - x^{-2})$	
			Stationary point $\rightarrow$ (their) $\frac{dy}{dx} = 0$ soi	M1		
			$x^2 = 1$ or $y^2 = 2$ or $y^4 = 4$	A1	Ignore any other values	
			$(1, \sqrt{2})$, $(1, -\sqrt{2})$	A1,A1	Accept 1.41 or $4^{\frac{1}{4}}$ for $\sqrt{2}$	SR. Award **A1** only if extra co-ordinates presented with both correct answers
				[7]		
4	(i)		Produce (at least 2) relevant equations	M1	e.g. $1 + 2\lambda = 6 + \mu$, $2 + \lambda = 8 + 4\mu$, $3\lambda = 1 - 5\mu$	
			Eliminate either λ or μ from 2 of them and solve for the other (μ or λ)	M1	soi by correct (λ, μ)	
			$\lambda = 2$ and $\mu = -1$ cao	A1	or e.g. $\lambda = 2$ from 2 different pairs	
			Check that $(\lambda, \mu) = (2, -1)$ satisfies all eqns	B1	This must be convincing. Check unusual arguments	Dep previous M1M1A1 earned
			P is (5, 4, 6) cao www	A1	Allow any reasonable vector notation	
				[5]		
4	(ii)		Using $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ -5 \end{pmatrix}$	M1	i.e. correct parts for direction vectors	
			Using $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ giving value $\frac{n}{\sqrt{a}\sqrt{b}}$	M1	for any 2 meaningful vectors in this question using meaningful scalar product & modulus	Expect $\frac{-9}{\sqrt{14}\sqrt{42}}$
			68.2°... (not 111.8…)	A1	or 1.19 (radians)	
				[3]		

Question			Answer	Marks	Guidance	
5	(i)		their $\dfrac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M1		
			$\dfrac{dy}{dx} = \dfrac{2\sin\theta}{3\cos\theta}$	A1		
			their $\dfrac{dy}{dx} = \dfrac{1}{2}$	M1		
			$\tan\theta = \dfrac{3}{4}$	A1	If $\tan\theta = \frac{3}{4}$ not seen, award this A1 only if coords are correct	
			$(3.8, -0.6)$ or $\left(\dfrac{19}{5}, -\dfrac{3}{5}\right)$ or $x = 3.8,\ y = -0.6$	A1 **[5]**		
5	(ii)		Manipulating equations into form $\sin\theta = f(x)$ and $\cos\theta = g(y)$ and then using $\sin^2\theta + \cos^2\theta = 1$	M1	<u>If part (ii) is attempted first, and then part (i), allow</u> B1 for obtaining $\dfrac{dy}{dx} = \dfrac{4(x-2)}{9(y-1)}$ M1 for equating their $\dfrac{dy}{dx}$ to $\dfrac{1}{2}$ A1 for obtaining $9y - 8x = -7$ M1 for eliminating x or y from above eqn... A1 for $(3.8, -0.6)$	<u>the following marks in part (i):-</u> ….and their Cartesian equation
			$\dfrac{(x-2)^2}{9} + \dfrac{(1-y)^2}{4} = 1$ oe www ISW Accept e.g. $\left(\dfrac{x-2}{3}\right)^2$ $4x^2 + 9y^2 - 16x - 18y - 11 = 0$	A1 **[2]**		

Question			Answer	Marks	Guidance	
6			Attempt diff to connect du & dx Correct result e.g. $\frac{du}{dx} = 2$ or $du = 2\,dx$	M1 A1	or find $\frac{du}{dx}$ or $\frac{dx}{du}$	
			Indef integ in terms of u = $\frac{1}{2}\int \frac{2u-3}{u^5}(du)$	A1	Must be completely in terms of u.	
			Integrate to $\frac{u^{-3}}{-3} - \frac{3u^{-4}}{-8}$ oe	A1A1	or (using 'by parts') $\frac{(2u-3)u^{-4}}{-8} - \frac{u^{-3}}{12}$	Award B1,B1 for $\frac{4u^{-3}}{-3} - \frac{3u^{-4}}{-2}$ or for $\frac{2u^{-3}}{-3} - \frac{3u^{-4}}{-4}$ or for $\frac{(2u-3)u^{-4}}{-2} - \frac{u^{-3}}{3}$ or for $\frac{(2u-3)u^{-4}}{-4} - \frac{u^{-3}}{6}$
			Use correct variable & correct values for limits	M1	Provided minimal attempt at $\int f(u)du$ made	
			= $\frac{-23}{384}$ oe (-0.059895 …..)	A1	Accept decimal answer only if minimum of first 3 marks scored	
			[ISW,e.g. changing to $\frac{23}{384}$]			
				[7]		

Question			Answer	Marks	Guidance	
7	(i)	I	$\frac{\cos x}{1+\sin x} - \frac{-\sin x}{\cos x}$ or $\frac{\cos x}{1+\sin x} + \frac{\sin x}{\cos x}$	B2	Each half (including 'middle' sign) scores B1	
			$\frac{+/-\cos^2 x +/- \sin x(1+\sin x)}{(1+\sin x)\cos x}$	M1	Combine, provided derivative was of form $f'(x)/f(x)$	Allow only variations num signs
			$\frac{1+\sin x}{\cos x(1+\sin x)} = \frac{1}{\cos x}$ www **AG**	A1	$\cos^2 x + \sin^2 x = 1$ in intermediate step required	
		II	Change to $\ln\left(\frac{1+\sin x}{\cos x}\right)$	B1		
			Change to ln(sec *x* + tan *x*)	B1	Not $\ln(\frac{1}{\cos x} + \tan x)$	
			Diff as $\frac{\text{attempt at } \frac{d}{dx}(\sec x + \tan x}{\sec x + \tan x}$	M1		
			Reduce to $\sec x = \frac{1}{\cos x}$	A1		
		III	Change to $\ln\left(\frac{1+\sin x}{\cos x}\right)$	B1		
			Diff as $\frac{\text{attempt at quotient differentiation}}{\frac{1+\sin x}{\cos x}}$	M1		
			Fully correct differentiation	A1		
			Correct reduction to $\frac{1}{\cos x}$	A1 **[4]**		
7	(ii)		Indef integral = $\ln(1+\sin x) - \ln(\cos x)$ [Method I]	B1	or $\ln(\sec x + \tan x)$ [Method II]	
			Substitute limits & use log manipulation	M1	Use of ln A – ln B = $\ln\frac{A}{B}$ anywhere in question	
			Answer = ln(2 + √3)	B1 **[3]**	Accept ln 3.73 or $\ln\frac{2+\sqrt{3}}{1}$ …..but not $\ln\frac{1+\sqrt{3}/2}{1/2}$	Answer has not been given

Question			Answer	Marks	Guidance	
8	(i)		$AB = \sqrt{(+/-2)^2 + (+/-2^2 + (+/-4)^2)}$ $AD = \sqrt{(+/-2)^2 + (+/-4)^2 + (+/-2)^2}$	B1 B1 **[2]**	oe oe	If $AB^2 = AD^2 = 24$, then SR B1 $AB = AD$ to be stated for 2nd B1
8	(ii)		midpoint is (3, 5, 0) Clear method for finding direction vector $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(3\mathbf{j} - \mathbf{k})$ oe or e.g. $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} + \mu(-3\mathbf{j} + \mathbf{k})$ cao	B1 M1 A1 **[3]**	Accept any reasonable vector notation. Expect $3\mathbf{j} - \mathbf{k}$ or $-3\mathbf{j} + \mathbf{k}$ **"r ="** is essential. No f.t. for wrong mid-point.	
8	(iii)		substitution of $\lambda = +/-5$ or $\mu = +/-4$	M1 **[1]**	Based on correct answer to (ii)	
8	(iv)		Kite	B1 **[1]**		

Question			Answer	Marks	Guidance	
9	(i)		Separating variables $\int \frac{1}{\theta+20}\,\mathrm{d}\theta = \int -k\,\mathrm{d}t$	M1	or invert each side: $\frac{\mathrm{d}t}{\mathrm{d}\theta} = -\frac{1}{k(\theta+20)}$	Must see $\frac{1}{\theta+20}$; ignore posn 'k'
			$\ln(\theta + 20) = -kt$ (+ c) or equivalent	A1	"Eqn A"	
			$\theta = A\mathrm{e}^{-kt} - 20$ oe (i.e. $\theta = \mathrm{e}^{-kt+c} - 20$)	A1	"Eqn B"	
				[3]		
9	(ii)		$(-)3 = -k(40+20)$	M1	Using $t = 0, \theta = 40, \frac{\mathrm{d}\theta}{\mathrm{d}t} = (-)3$ in <u>given</u> equation	
			$k = \frac{1}{20}$ oe	*A1	Not $k = -\frac{1}{20}$	
			Subst $t = 0, \theta = 40$ & their k (where necessary) into their Eqn A or their Eqn B and solve for the arbitrary constant	M1		
			Subst $\theta = 0$ & their values of k and the arbitrary constant into their Eqn A or their Eqn B	M1		
			t = 21.9722 = 22 minutes cao www	dep*A1		
				[5]		
9	(iii)		k is larger	B1		
				[1]		

Question			Answer	Marks	Guidance	
10	(i)		Clear start to algebraic division (Quotient) = $x-1$ (Remainder) = $x+7$ Final answer: $x-1+\frac{x+7}{x^2-x-6}$	M1 A1 A1 A1 **[4]**	at least as far as x term in quot & subseq mult back final answer in correct form This must be shown in part (i) or, if not, then implied in part (ii) If no long division shown but only comparison of coefficients or otherwise, SR M0 B1 B1 B1	& attempt at subtraction Accept $A=1, B=-1, C=1, D=7$
10	(ii)		Convert their $\frac{Cx+D}{x^2-x-6}$ to Partial Fracts $\frac{x+7}{x^2-x-6} = \frac{2}{x-3}-\frac{1}{x+2}$ Their …. $\int Ax+B\,\mathrm{d}x=\frac{1}{2}Ax^2+Bx$ or $\frac{(Ax+B)^2}{2A}$ $\int \frac{E}{x-3}+\frac{F}{x+2}\,\mathrm{d}x = E\ln(x-3)+F\ln(x+2)$ Using limits in a correct manner $8+\ln\frac{27}{4}$ $\left(8+\ln\frac{54}{8}\right)$ isw	M1 A1A1 B1 ft B1 ft M1 A1 **[7]**	 Correct fraction converted to correct PFs Tolerate some wrong signs provided intention clear Answer required in the form $a+\ln b$, so giving only a decimalised form is awarded A0	

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